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Don't forget your students' growth in and use of geometry each year!

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## References

Battista, Michael T. "Learning with Understanding: Principles and Processing in the Construction of Meaning for Geometric Ideas." In *The Learning of Mathematics Sixty-ninth Yearbook*, edited by Martin and Strutchens. NCTM, 2007.

For additional reading, read Glenda Lappan's (NCTM President, 1998–2000) President's Message, "Geometry: the Forgotten Strand" at www.nctm.org/ about/content.aspx?id=996.

## DON'T FORGET GEOMETRY!

Focusing on setting the stage for algebra in pre-K–grade 8 math instruction has recently been a major topic of public discussion. However, over the last two decades of international studies, U.S. students continue to perform more poorly on international comparisons in geometry than they do in number, algebra, or data. As you finish this year and prepare curriculum for next year, I challenge you—don't forget your students' growth in and use of geometry each year!

The study of geometry is about understanding the spatial world in which we live—thinking about how things look, how they are arranged, where they are, and how they move. Geometry is important in its own right, but it is also critical for the field of mathematics. For instance, consider how arrays help us understand multiplication and prime numbers, how similarity helps us understand proportionality, and how graphing helps us reason about functions, continuity, differentiation, and integration.

Our students' learning of geometry must take them well beyond naming shapes and accumulating formulas. Recognition and differentiation of shapes starts a neverending study of the properties *same*, *similar*, and *symmetric* in objects throughout mathematics. Young children experience two-dimensional representations in pictures of familiar 3-D objects. Use of these representations continues into study of the arts, crystallography, DNA, and architecture. A major current application of these ideas is computer graphics, where they play critical roles in video games, fantasy games, and projective views of museum galleries on the Web.

One of the most useful guides that teachers have for constructing a strong geometry content curriculum is van Hiele's theory of successive levels of sophistication in students' geometric thinking (Battista 2007). This theory describes learners' experiences with geometry at different stages and explains their thinking at each stage on the basis of their earlier understanding of relationships. At the first level, their thinking is based primarily on holistic images. With appropriate instructional guidance, students gradually move to a descriptive-analytic level, in which they use formal concepts like congruence and perpendicularity to characterize shapes by their properties. At the third level—the relational-inferential level—students' reasoning grows to include using properties, definitions, and inference to understand relationships among classes of shapes (e.g., between squares and rectangles) and among properties of individual shapes.

With proper instructional guidance, students' reasoning progresses from vague visual impressions to empirical exploration to deduction. Without a strong background in these three levels of thinking, students are likely to experience frustration or failure in high school geometry, which challenges them to move to deductive reasoning. We must provide students with experiences that stimulate their curiosity. Geometry offers an opportunity for students to engage in mathematical thinking that allows them to make conjectures. It initiates reasoning and opens the door to developing deductive reasoning and an understanding of proof.

Be sure to offer your students opportunities to view geometry from several perspectives, including synthetic approaches, but also transformations, coordinate systems, and vectors. In this way, you can build the rich, deep geometry background that is necessary for a correspondingly deep study of topics in a traditional Algebra 2 curriculum and beyond.

Prepare your students to use their geometric reasoning in problem-solving settings, routinely making connections to algebraic ideas. Through these experiences, they will learn that presenting their own reasoning and evaluating that of others is the essence of "doing mathematics." In the process, they will also learn that geometry is an integral part of mathematical thinking and problem solving.  $\Omega$ 

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